attenuation factor; F_{sp}, specific surface of disperse material, $F_{sp}=3/(r_p\rho_p)$; μ, mass concentration of particles in the stream, $\mu=G_p/(v_pF_2)$; c_0, coefficient of release of radiation by an absolute blackbody, $c_0 = 5.67\ W/(m^2\cdot{}^\circ K^4)$; N, number of particles in the stream; A_p, absorptivity of the disperse stream; $\varphi_{2,1}$, exposure factor, $\overline{\varphi_{2,1}}=F_1/(\pi h^2)$; Q, amount of thermal energy; ε, degree of blackness of the system; ε_{rd}, reduced degree of blackness of the system; θ, generalized temperature. Subscripts: m, gaseous medium; p, polymer; sur, particle surface; c, center of particle; fu, fusion; abl, ablation; sou, source; absn, absorption; fli, flight; ons.fu, onset of fusion; c.fu, complete fusion.

LITERATURE CITED

1. A. Hasui, The Spray-Coating Technique [Russian translation], Mashinostroenie, Moscow (1975).
2. A. V. Donskoi and V. S. Klubnikin, Electroplasma Processes and Installations in Engineering [in Russian], Mashinostroenie, Moscow (1979).
3. D. A. Rodchenko, M. I. Petrokovets, and A. I. Barkan, "Special features of the heating of disperse polytetrafluorethylene in a low-temperature plasma jet," Vestsi AN BSSR. Ser. Fiz.-Tekh. Navuk, No. 4, 52-56 (1983).
4. A. V. Lykov, The Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).
5. M. A. Mikheev, Fundamentals of Heat Transfer [in Russian], Gosénergoizdat, Moscow (1956).
6. V. P. Isachenko, V. A. Osipova, and A. S. Sukomel, Heat Transfer [in Russian], Énergiya, Moscow (1981).
7. S. S. Kutateladze, Fundamentals of the Theory of Heat Exchange [in Russian], Nauka, Novosibirsk (1970).
8. L. N. Novichenok and Z. P. Shul'man, Thermophysical Properties of Polymers [in Russian], Nauka i Tekhnika, Minsk (1971).
9. T. A. Speranskaya and L. I. Tarutina, Optical Properties of Polymers [in Russian], Khimiya, Leningrad (1976).

PROPAGATION OF SOUND PERTURBATIONS IN HETEROGENEOUS GAS—LIQUID SYSTEMS

I. Sh. Akhatov and V. A. Baikov

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It is shown that the spatial heterogeneity of the gas content and the disperson of sound in a bubble-containing medium may lead to deviation, focusing, and defocusing of sound beams.

Analysis of the processes that occur during the passage of pressure waves through a gas—liquid mixture having a bubble structure is required for solving problems of energetics and pipeline transportation. Wave propagation in gas—liquid media was investigated in [1, 2] in the approximation of plane one-dimensional motion. However, experimental data on gas—liquid flows in pipes suggest that the parameters of the mixture (for example, the spatial gas content [3, 4]) are not homogeneous over a cross section of the pipe. The present work is devoted to revealing the features of the propagation of the sound perturbations in heterogeneous gas—liquid media.

The equations of continuity and impulse for a single-velocity gas—liquid mixture are of the form

$$\frac{\partial\rho}{\partial t}+\operatorname{div}(\rho\mathbf{v})=0,\ \rho=\rho_1^0\alpha_1+\rho_2^0\alpha_2,\ \alpha_1+\alpha_2=1,$$
$$\rho\frac{\partial\mathbf{v}}{\partial t}+\rho(\mathbf{v}\nabla)\mathbf{v}=-\nabla p,\ p=\alpha_1p_1+\alpha_2p_2. \tag{1}$$

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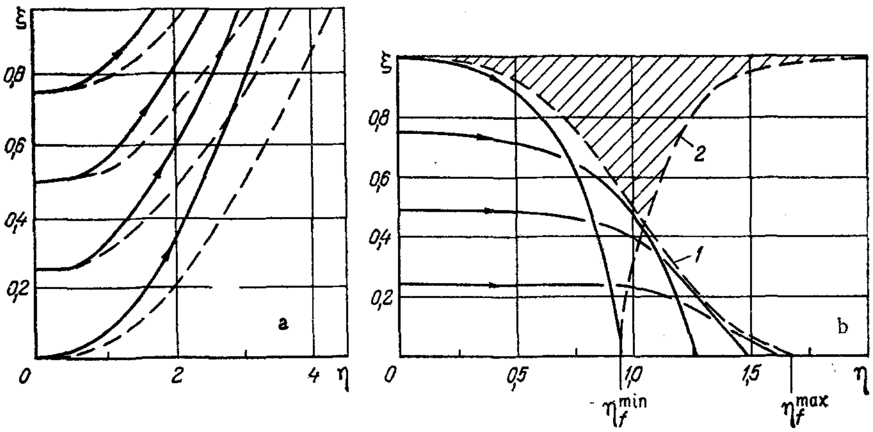

Fig. 1. Beam trajectory for the linear (a) and parabolic (b) gas content distributions along the ξ axis.

We assume that the liquid phase is incompressible (ρ°_1 = const). This assumption holds true for reasonably large values of the gas content ($\alpha_2 \gg \alpha^*_2 = p/\rho^\circ_1 c^2_1$, c_1 is the velocity of sound in a pure liquid). Let us consider a locally monodispersive medium, elementary volumes of which contain spherically shaped bubbles of equal radii. We take the Rayleigh–Lamb equation [5], describing the radial motion of the wall of a single bubble in a viscous incompressible liquid, as the equation of the state of the mixture.

$$R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2=\frac{p_2-4\mu_1\dfrac{dR}{dt}-p_1}{\rho_1^0}. \qquad (2)$$

We take the equation of state of the gas inside the bubble in the form

$$p_2=p_0\left(\frac{\rho_2^0}{\rho_{20}^0}\right)^\gamma. \qquad (3)$$

For small spatial concentrations of the gas ($\alpha^*_2 \ll \alpha_2 \ll 1$), the entire mixture can be considered as a continuous medium with density approximately equal to the reduced density of the liquid ($\rho = \rho^\circ_1\alpha_1$), pressure equal to the true pressure in the liquid phase ($p \approx p_1$), and compressibility equal to that of the gas.

Let us assume that the perturbations of pressure ($p' = p - p_0$), density ($\rho' = \rho - \rho_0$), velocity of the mixture, as well as oscillations of a bubble ($R' = R - R_0$) are small. From now on, the subscript 0 will designate the parameters of an unperturbated flow.

Assuming that bubbles are neither destroyed nor produced in the mixture, it can readily be shown that the perturbations ρ' and R' are related by

$$\rho'=-\frac{3\alpha_{10}\alpha_{20}\rho_1^0}{R_0}R'. \qquad (4)$$

Equations (1)-(3) after linearization and taking account of (4) can be reproduced to a single equation for ρ':

$$\frac{\partial^2\rho'}{\partial t^2}=\Delta\left(c^2\rho'+\alpha\frac{\partial\rho'}{\partial t}+\beta\frac{\partial^2\rho'}{\partial t^2}\right),\quad \Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2},$$
$$c^2=\frac{\gamma p_0}{\alpha_{10}\alpha_{20}\rho_1^0},\quad \alpha=\frac{4\mu_1}{3\alpha_{10}\alpha_{20}\rho_1^0},\quad \beta=\frac{R_0^2}{3\alpha_{10}\alpha_{20}}, \qquad (5)$$

where c is the equilibrium (low-frequency) velocity of sound in the bubble medium. If the spatial gas content α_{20} is heterogous in space, then the coefficients c, α and β, in (5) depend on the spatial coordinates. It is easy to show that for sufficiently long perturbations $\lambda \gg \max(\mu_1/\alpha_{20}c\rho^\circ_1, R_0/\sqrt{\alpha_{20}})$ the terms $\alpha\partial\rho'/\partial t$ and $\beta\partial^2\rho'/\partial t^2$ are small in comparison with $c^2\rho'$. Therefore, α and β can be considered as constants that correspond to a certain mean value α°_{20}, and c varies spatially according to the spatial gas content.

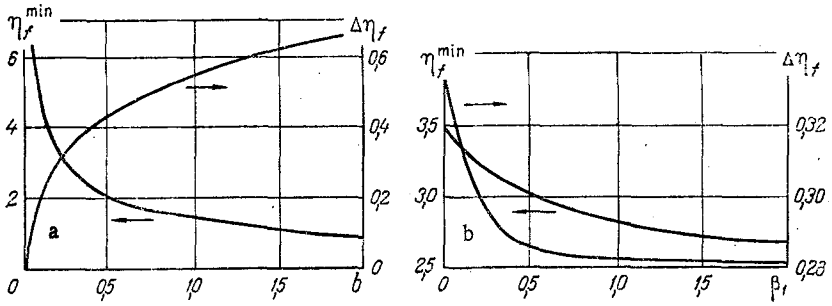

Fig. 2. Dependence of the focal distance η_f^{min} and aberration $\Delta\eta_f$ on the parameter of heterogeneity of the medium b (a) and parameter of dispersion β_1 (b).

Let us assume that the gas content at distances of the order of a wavelength changes only slightly. In this case the approximation of geometrical optics [6] can be used for solving (5). Then for a harmonic wave of the form $\rho' = (u(r)/c^2(r))\exp(-i\omega t)$ and (5) we will obtain Helmholtz's equation

$$\Delta u + (k_1^2(r) + i\delta(r))u = 0,$$
$$k_1^2(r) = k_0^2 \frac{c_0^2}{c^2}\left(1 + \beta_1 \frac{c_0^2}{c^2}\right), \quad k_0 = \frac{\omega}{c_0} = \frac{2\pi}{\lambda_0}, \tag{6}$$
$$\delta(r) = \frac{\alpha k_0^3 c_0^3}{c^4}, \quad \beta_1 = k_0^2 \beta,$$

where $c_0 = c(\alpha^\circ_{20})$ is a typical value of the velocity of sound, and β_1 is a dimensionless parameter characterizing the dispersive properties of the medium.

Representing the solution in the form $u(r) = A(r)\exp(ik_0\psi(r))$, we obtain the eikonal equation for the eikonal $\psi(r)$ and the transfer equation for the amplitude $A(r)$

$$(\nabla\psi)^2 = n^2(r), \quad n^2(r) = \frac{k_1^2(r)}{k_0^2}, \tag{7}$$

$$A\Delta\psi + 2\nabla\psi\cdot\nabla A + \delta_1(r)A = 0, \quad \delta_1(r) = \frac{\delta(r)}{k_0}. \tag{8}$$

Here n is the refractive index of the medium which, for the case in question, depends not only on the spatial coordinates but also on the wavelength. From (8) it follows that the intensity of absorption of the energy of a sound wave is equal to $\delta_1 A^2$ per unit volume. Estimates show that the absorption is negligible for the gas—liquid systems considered in the present work.

Let us consider the propagation of sound perturbations in a channel filled with a gas—liquid medium in which the gas content depends on the z-coordinate only. From (7) it follows that in this case, the equation for the ray trajectors (i.e., for the line perpendicular to the surfaces $\psi(r)$ = const) emanating from the point with coordinates $z = z_0$, $x = 0$ at an angle $\pi/2$ to the z axis is of the form

$$x = \int_{z_0}^{z} \frac{n(z_0)\,dz}{\sqrt{n^2(z) - n^2(z_0)}}. \tag{9}$$

For a linear distribution of the gas content along the z axis ($0 \le z \le a$) $\alpha_{20}(z) = \alpha^\circ_{20}\left(1 + d\frac{z}{a}\right)$, which can hold, for example, as the gas—liquid mixture becomes stratified in the gravitational field, the equation for the trajectories of rays (9) can be written as

$$\xi + \xi_* = (\xi_0 + \xi_*)\,\mathrm{ch}(\varkappa\eta), \quad \xi = z/a, \quad \eta = x/a,$$
$$\xi_* = \frac{1 + 2\beta_1}{2\beta_1 d}, \quad \varkappa = \frac{\sqrt{\beta_1}\,d}{n(\xi_0)\,a}, \quad \xi_0 = z_0/a,$$
$$n^2(\xi) = 1 + \beta_1 + d(1 + 2\beta_1)\xi + \beta_1 d^2\xi^2.$$

Evidently, the trajectories of the rays depend only on the two dimensionless constants d and β_1 which characterize the heterogeneity and dispersion of the medium, respectively. In Fig. 1a, the calculated trajectories of rays emanating from the points ξ_0 = 0, 0.25, 0.5 and 0.75 are shown for d = 0.2 and the two different values of the parameter β_1 (β_1 = 2.77 for the solid lines, β_1 = 0.11 for the dashed curves). These values of β_1 correspond, for example, to the propagation of waves of different wavelengths $\lambda_0 = 10^{-2}$ m (β_1 = 2.77) and $\lambda_0 = 5 \cdot 10^{-2}$ m (β_1 = 0.11) in the gas–liquid mixture in which α°_{20} = 0.05, $R_0 = 10^{-3}$ m. As is seen from Fig. 1, the trajectories of rays are bent in the direction of increasing spatial gas content, the curvature being increased with increasing dispersion parameter β_1. The calculations also showed an increasing curvature of the rays with increasing parameter of heterogeneity d.

For a parabolic distribution of the gas content along the z axis ($|z| \le a$) $\alpha_{20}(z) = \alpha^\circ_{20} \left(1 + b\left(1 - \frac{z^2}{a^2}\right)\right)$, which can take place, for example, near the wall as the gas–liquid mixture flows along the pipe [3, 4], the equation for the trajectory of the rays (9) is of the form

$$\eta = B\left[\frac{\pi}{2} F\left(\frac{1}{2}, \frac{1}{2}; 1; \frac{\xi_0^2}{\Delta}\right) - \int\limits_0^{\arcsin\frac{\xi}{\xi_0}} \frac{dt}{\sqrt{1 - \frac{\xi_0^2}{\Delta}\sin^2 t}}\right],$$

$$\Delta = 2 + \left(2 + \frac{1}{\beta_1}\right)\frac{1}{b} - \xi_0^2,\quad B = \frac{n(\xi_0)}{\sqrt{\beta_1 \Delta b}},$$

$$n^2(\xi) = 1 + \beta_1 + b(1 + 2\beta_1)(1 - \xi^2) + \beta_1 b^2 (1 - \xi^2)^2,$$

where F(a, b; c; z) is Gauss's hypergeometric function.

In Fig. 1b, the trajectories of the rays emanating from the points ξ_0 = 0.25, 0.5, 0.75 and 1 are shown when the parameter of heterogeneity of the medium b = 2 and the parameter of dispersion β_1 = 0.2. Figure 1b demonstrates the presence of aberration. The rays, depending on the initial coordinate ξ_0 intersect the x axis at different points $\eta_f(\xi_0)$

$$\eta_f(\xi_0) = \frac{\pi}{2} BF\left(\frac{1}{2}, \frac{1}{2}; 1; \frac{02}{\Delta}\right),\quad \eta_f^{\max} = \eta_f(0),\quad \eta_f^{\min} = \eta_f(1).$$

The beam $|\xi_0| \le 1$ is focused on the segment $[\eta_f^{\min}, \eta_f^{\max}]$. The dashed curve 1 shows the envelope of the family of trajectories of the rays. There is a region of "shadow" where the sound perturbations do not penetrate; this region is bounded by the envelope and a ray emanating from the point ξ_0 = –1 (dashed curve 2).

In Fig. 2a the calculated dependences for $\eta_f^{\min}$ and $\Delta\eta_f = \eta_f^{\max} - \eta_f^{\min}$ on the parameter of heterogeneity of the medium b at β_1 = 0.2 are shown. As is seen from Fig. 2, the value of the parameter b considerably influences the focal distance η_f and the value of the aberration $\Delta\eta_f$. As b diminishes to zero, η_f asympotically approaches infinity, and $\Delta\eta_f$ approaches zero, since the case b = 0 corresponds to a homogeneous medium.

Figure 2b shows the calculated dependencies for $\eta_f^{\min}$ and $\Delta\eta_f$ on the parameter of dispersion β_1 at b = 0.2. It can be seen that β_1 affects the value of the focal distance only slightly and does not practically affect the value of the aberration. As β_1 diminishes to zero, $\eta_f^{\min}$ and $\Delta\eta_f$ approach their limiting values

$$\eta_f^{\min} = \frac{\pi}{2\sqrt{b}},\quad \eta_f^{\max} = \frac{\pi}{2\sqrt{b}}(\sqrt{1+b} - 1).$$

It should be noted that for b < 0 the sound beam is defocused.

NOTATION

v, Velocity; ρ°_i, true density; p_i, pressure; α_i, spatial concentration; i, phase number; R, bubble radius; μ_1, viscosity of the liquid; γ, polytropic index; ω, frequency; c, velocity of sound; k, wave number; ψ, eikonal; A, amplitude; n, index of refraction; β, dispersion parameter; δ, absorption coefficient; λ, wavelength; x, z, ξ, η, coordinates; a, beam width; d, b, parameters of heterogeneity. Indices: 1, liquid phase; 2, gas phase.

LITERATURE CITED

1. A. A. Gubaidullin, A. I. Ivandaev, and R. I. Nigmatulin, "Nonstationary waves in a liquid containing gas bubbles," Dokl. Akad. Nauk SSSR, 226, No. 6, 1299-1302 (1976).
2. V. E. Nakoryakov, V. V. Sobolev, and I. R. Shreiber, "Long-wave perturbations in a gas—liquid mixture," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5, 71-76 (1972).
3. V. E. Nakoryakov, "Hydrodynamics of two-phase flows," in: Hydrodynamics and Heat Exchange in Two-Phase Media [in Russian], ITF Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1981), pp. 5-30.
4. D. Batterworse and G. Hewitt (eds.), Two-Phase Flow and Heat Transfer [Russian translation], Énergiya, Moscow (1980).
5. R. I. Nigmatulin, Foundations of Mechanics of Heterogeneous Media [in Russian], Nauka, Moscow (1978).
6. I. V. Vinogradova, O. V. Rudenko, and A. P. Sukhorukov, Wave Theory [in Russian], Nauka Moscow (1979).

INFLUENCE OF PHYSICAL AND SCHEMATIC VISCOSITY IN THE ANALYSIS OF THE NEAR WAKE BEHIND A DISC

I. A. Belov, S. A. Isaev,
M. I. Nisht, and A. G. Sudakov

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The influence of the physical and schematic viscosity on the results of computations performed within the framework of viscous and ideal medium models is analyzed in an example of uniform incompressible fluid flow around a disc.

1. The development of the near wake behind a disc is a typical example of the flow around bodies with a fixed site of flow separation on their surface. Investigation of the influence of viscosity in computing such flows is of important value in the determination of integral and local characteristics of poorly streamlined bodies, particularly the base pressure. Interest is also stimulated in the computation of separation flows by the practical utilization of flow control principles because of the premeditated formation of developed circulation zones near the streamlined bodies (see [1], for example). Separation flows can be modelled correctly on the basis of the solution of the system of complete nonstationary Navier—Stokes equations. The complexity of realizing such an approach even in investigations of stationary separation flows for significant Reynolds numbers is well known. The initial system of time-averaged Navier—Stokes equations in Reynolds form is not closed in this case and requires reliance on semiempirical models of turbulence which describe such flow structural elements of different scale as thin shear layers and vortex formations, whose dimensions are commensurate with the size of the body being streamlined.

In a number of cases, particularly in the solution of problems about the flow around a body with sharp edges at ultimately high Reynolds numbers when the influence of molecular viscosity on the flow becomes insignificant, the difficulties in solving the complete Navier—Stokes or the Reynolds equations resulted in the development of methods to compute separation flows that are based on a model of an ideal medium [2]. Let us mention just two, the method of discrete vortices and the method of coarse particles, whose detailed description is given in [3] and [4], respectively. The satisfactory agreement between the computed results obtained by using these methods and the experimental data (on body drag, flow configuration, etc.) affords a basis for the assumption that such an approach is justified in the consideration of fully developed turbulent flows. Let us note that modelling the turbulence is here presumably associated with singularities in the numerical realization of the analysis that

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